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SHOCK WAVES IN A PLASTIC COMPACTING MEDIUM

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The problem of slow propagation of a plastic deformation was solved by S. L. Sobolev.[1] We shall consider a shock propagation of a plastic deformation in a medium which is characterized by the following properties. In the initial state its density is ρ_0, at this density it exerts a negligibly small resistance to compression. When brought to a density ρ_1, the medium is incompressible and in this state is plastic, whereupon it is assumed that the absolute value of the most "tangential" tension is linearly dependent on the average normal tension (the plasticity conditon of Prandtl)[2]. This kind of plasticity, for example, can be possessed by sand.

A shock wave emerges (is formed) under the effect of an explosion in some very small spherical volume with radius R_0. The main axes of the stress tensor coincides with the coordinate lines of the circle system, whose center is superposed (congruent) with the center of explosion. The main normal tensions we shall name as σ_r and $\sigma_0 = \sigma_\phi$. The condition of plasticity, according to the assumption, must be written thus:

$$\sigma_r - \sigma_0 = k + m(\sigma_r + 2\sigma_0) \tag{1}$$

The equations of motion of the compacted medium have the form

$$\rho_1\left(\frac{\partial u}{\partial t} + \frac{u\partial u}{\partial r}\right) = \frac{\partial \sigma_r}{\partial r} + \frac{2(\sigma_r - \sigma_0)}{r}, \tag{2}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u) = 0. \tag{3}$$

From Eq. (3) it follows that

$$u = \frac{\lambda(t)}{r^2}. \tag{4}$$

Substituting this in (2) and eliminating σ_θ with the help of (1), we derive after integration:

$$\sigma_r = -\frac{k}{3m} + \rho_1\frac{\dot\lambda}{(\alpha-1)r} - 2\rho_1\frac{\lambda^2}{(\alpha-4)r^4} + C(t)\,r^{-\alpha}, \tag{5}$$

where

$$\alpha = \frac{6m}{1+2m}. \tag{6}$$

The boundary conditions on the front of the shock wave have the form

$$\rho_0\dot R = \rho_1(\dot R - u(R)), \tag{7}$$

$$\rho_0\dot R^2 = \rho_1(\dot R - u(R))^2 - \sigma_r(R). \tag{8}$$

Here R is the radius of the shock wave front.

At the boundary of the expanding cavity one must equate the pressure to the normal tension, taken with the opposite sign:

$$p = -\sigma_r(R_0). \tag{9}$$

The radius of the cavity is easily determined by starting with conservation of mass. If R_0 is the initial radius of the cavity, we derive

$$R_0 = \left[\left(1-\frac{\rho_0}{\rho_1}\right)R^3 + \frac{\rho_0}{\rho_1}R_0^3\right]^{1/3} \cong R\left(1-\frac{\rho_0}{\rho_1}\right)^{1/3}. \tag{10}$$

One can disregard the quantity R_0^3 here, even if $1 - \rho_0/\rho_1$ is of the order of several percent.

Assuming that the substance in the cavity expands like an ideal gas with an isentrope index γ, we find the expression for p:

$$p = p_0\left(\frac{R_0}{R}\right)^{3\gamma}\left(1-\frac{\rho_0}{\rho_1}\right)^{-\gamma}. \tag{11}$$

Excluding C(t) and $\lambda(t)$ from (7), (8) and (9), we come to the differential equation:

$$R\ddot R + \dot R^2\left\{2 + \frac{\rho_0}{\rho_1}\frac{\alpha-1}{1-\left(1-\frac{\rho_0}{\rho_1}\right)^{\frac{\alpha-1}{3}}} - 2\frac{(\alpha-1)\left[\left(1-\frac{\rho_0}{\rho_1}\right)^{\frac{4-\alpha}{3}}-1\right]}{(\alpha-4)\left[\left(1-\frac{\rho_0}{\rho_1}\right)^{\frac{1-\alpha}{3}}-1\right]}\right\} =$$

$$= \left\{\frac{\rho_1}{\alpha-1}\left(1-\frac{\rho_0}{\rho_1}\right)^{1/3}\left[\left(1-\frac{\rho_0}{\rho_1}\right)^{\frac{1-\alpha}{3}}-1\right]\right\}^{-1}\cdot\left\{p + \frac{k}{3m}\left[\left(1-\frac{\rho_0}{\rho_1}\right)^{-\frac{\alpha}{3}}-1\right]\right\}. \tag{12}$$

It is integrated in quadratures. Before writing down the quadrature, it is convenient to study the expression in the right hand side. The equation must, in any case, permit a solution which answers to the equilibrium, R = 0, $\dot R = 0$. In the right hand side of (12) there will stand 0, if

$$\left(\frac{R_0}{R}\right)^{3\gamma} = -\frac{k}{p_0}\frac{1}{3m}\left[\left(1-\frac{\rho_0}{\rho_1}\right)^{-\frac{\alpha}{3}}-1\right]\left(1-\frac{\rho_0}{\rho_1}\right)^{\gamma}. \tag{13}$$

Equilibrium must be achieved also then when $m = 0$, just as in the plasticity theory of Sen-Venana, because k and m are not connected with each other. But in this case it is necessary for equilibrium that k be less than zero. The ratio $|k|/p_0$ is a very small number, of the order of 10^{-4} or less. This justifies the approximation, made in (10), applicable to the condition of equilibrium (13) for not very large values of σ.

The condition of equilibrium (13) for $k < 0$ is satisfied for all values $m \geq -1/2$. But if $-1/2 \leq m < 0$, then the equilibrium radius is greater than for $m = 0$. This is not very probable, because m describes the additional (auxiliary) friction in the condition of Prandtl. Therefore, one must consider that $m < 0$. For very large values of m the equilibrium radius again becomes greater than for $m = 0$. Consequently, these values of m lie beyond the limits of applicability of the linear formula (1) in the problem under consideration.

Let us now introduce the following dimensionless quantities:

$$\frac{R}{R_0} = x; \quad \dot{R}\sqrt{\frac{\rho_0}{p_0}} = y; \quad 1 - \frac{\rho_0}{\rho_1} = \xi; \tag{14}$$

$$\mu = 4 + \frac{2(1-\xi)(\alpha-1)}{1-\xi^{\frac{\alpha-1}{3}}} - \frac{2(\alpha-1)\left(\xi^{\frac{1-\alpha}{3}}-1\right)}{(\alpha-4)\left(\xi^{\frac{1-\alpha}{3}}-1\right)}; \tag{15}$$

$$A = 2\,\frac{(\alpha-1)(1-\xi)}{\xi^{1/3}\left(\xi^{\frac{1-\alpha}{3}}-1\right)}. \tag{16}$$

In these variables the Eq. (12) is rewritten thus:

$$x\frac{d(y^2)}{dx} + \mu y^2 = A\left(\frac{x^{-3\gamma}}{\xi^{\gamma}} - \frac{|k|}{p_0}\frac{\xi^{-\frac{\alpha}{3}}-1}{3m}\right). \tag{17}$$

Its first integral is

$$y^2 = \frac{1-\xi}{\xi}x^{-\mu} + Ax^{-\mu}\int_1^x x^{\mu-1}\left(\frac{x^{-3\gamma}}{\xi^{\gamma}} - \frac{|k|}{p_0}\frac{\xi^{-\frac{\alpha}{3}}-1}{3m}\right)dx. \tag{18}$$

The approximation (10), and thus all those following, is justified only in the case if in the integral (18) the region close to 1 does not give a material

contribution, i.e., if $\mu - 3\gamma > 0$. This condition does not seem particularly restrictive, as evident from Table 1 of the values μ for different α and porosity $\xi = 1 - \rho_0/\rho_1$.

Table 1

ξ \ α	0	1	2	3
0,01	3,26	4,44	6,00	7,84
0,1	3,78	4,78	5,98	7,40
0,2	4,12	5,00	5,96	7,12

Considering that condition $\mu - 3\gamma > 0$ is satisfied, one can disregard also the first term in Eq. (18), of which one is easily convinced by comparing the coefficient A/ξ^{γ} with the quantity $(1-\xi)/\xi$. Then a simple expression is derived for y^2:

$$y^2 = A\left(\frac{x^{-3\gamma}}{(\mu-3\gamma)\xi^{\gamma}} - \frac{|k|}{p_0}\frac{\xi^{-\frac{\alpha}{3}}-1}{3m\mu}\right), \tag{19}$$

from which there is obtained the value of the maximum radius of wave expansion

$$R_m = R_0\left[\frac{3m\mu}{\mu-3\gamma}\,\frac{1}{\left(\xi^{-\frac{\alpha}{3}}-1\right)\xi^{\gamma}}\,\frac{p_0}{|k|}\right]^{\frac{1}{3\gamma}}. \tag{20}$$

The substance remains compressed even after load release, since the compression is irreversible. R_m is larger than the equilibrium radius in the ratio $[\mu/(\mu-3\gamma)]^{1/3\gamma}$.

The time of full expansion is equal to

$$t_m = R_0\sqrt{\frac{\rho_0}{p_0}\,\frac{(\mu-3\gamma)\,\xi^{\gamma}}{A}\left(\frac{R_m}{R_0}\right)^{\frac{3\gamma+2}{2}}}\;\frac{\Gamma\left(\frac{1}{3\gamma}+\frac{1}{2}\right)\sqrt{\pi}}{\Gamma\left(\frac{1}{3\gamma}+1\right)}. \tag{21}$$

Note that there did not go into the problem any relationship between the tensions and the deformations or the deformation velocities. Here we find an analogy with the statically determinable flat problems of the plasticity theory.

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1. S. L. Sobolev. Trudy of seismological instr. 49 (1935).
2. L. S. Leibenzon. Elements of mathematical theory of plasticity, 1943, page 56.